分布定数回路（2）　《半無限長線路》

半無限長線路では反射波がないので、①,②より$B = 0$として、

$$\begin{cases} \dot{V}(x) = Ae^{-\dot{\gamma}x} & \cdots③ \\ \dot{I}(x) = \dfrac{A}{\dot{Z}_0}e^{-\dot{\gamma}x} & \cdots④ \end{cases}$$

③,④より $\dfrac{\dot{V}(x)}{\dot{I}(x)} = \dfrac{Ae^{-\dot{\gamma}x}}{\dfrac{A}{\dot{Z}_0}e^{-\dot{\gamma}x}} = \dot{Z}_0 \quad \cdots⑤$

⑤より線路の任意の点において、入力インピーダンスは$\dot{Z}_0$

⑥,⑦より $\dot{V}(0) = E - \dfrac{\dot{Z}}{\dot{Z}_0}\dot{V}(0) \quad \cdots⑧$

③に$x = 0$を代入　$\dot{V}(0) = A \quad \cdots⑨$

⑧に⑨を代入　$A = E - \dfrac{\dot{Z}}{\dot{Z}_0}A$　　$A = \dfrac{\dot{Z}_0\dot{E}}{\dot{Z} + \dot{Z}_0} \quad \cdots⑩$

③,④に⑩を代入

$$\begin{cases} \dot{V}(x) = \dfrac{\dot{Z}_0\dot{E}}{\dot{Z} + \dot{Z}_0}e^{-\dot{\gamma}x} \\ \dot{I}(x) = \dfrac{\dot{E}}{\dot{Z} + \dot{Z}_0}e^{-\dot{\gamma}x} \end{cases}$$

伝搬方程式の解

入射波　反射波

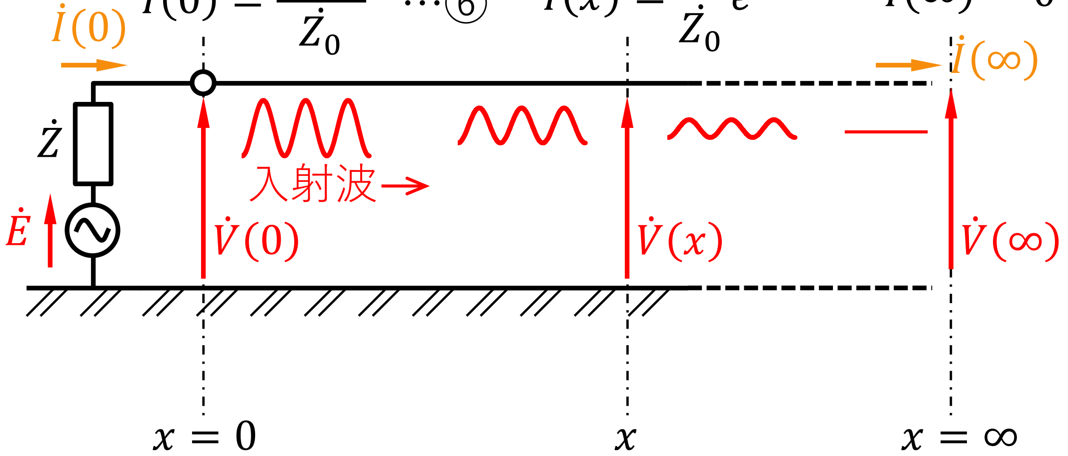
$$\begin{cases} \dot{V}(x) = Ae^{-\dot{\gamma}x} + \sout{Be^{\dot{\gamma}x}} & \cdots① \\ \dot{I}(x) = \dfrac{1}{\dot{Z}_0}\left(Ae^{-\dot{\gamma}x} \sout{- Be^{\dot{\gamma}x}}\right) & \cdots② \end{cases}$$

伝搬定数：$\dot{\gamma} = \sqrt{(R + j\omega L)(G + j\omega C)}$

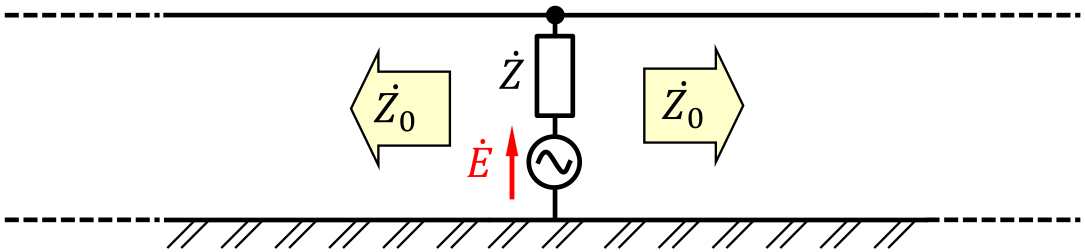
特性インピーダンス：$\dot{Z}_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$\dot{V}(0) = E - \dot{Z}\dot{I}(0) \quad \cdots⑦$　　$\dot{V}(x) = Ae^{-\dot{\gamma}x}$　　$\dot{V}(\infty) = 0$

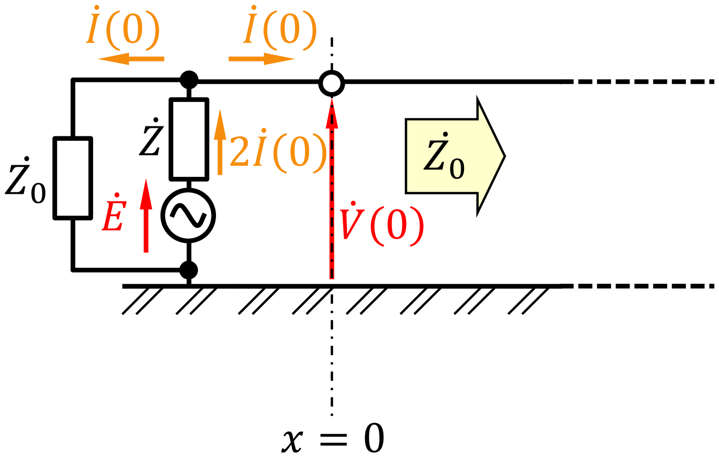
$\dot{I}(0) = \dfrac{\dot{V}(0)}{\dot{Z}_0} \quad \cdots⑥$　　$\dot{I}(x) = \dfrac{A}{\dot{Z}_0}e^{-\dot{\gamma}x}$　　$\dot{I}(\infty) = 0$



分布定数回路（2）　《両無限長線路》



等価変換



伝搬方程式の解

入射波　反射波

$$\begin{cases} \dot{V}(x) = Ae^{-\dot{\gamma}x} + \sout{Be^{\dot{\gamma}x}} & \cdots ① \\ \dot{I}(x) = \dfrac{1}{\dot{Z}_0}\left(Ae^{-\dot{\gamma}x} \sout{- Be^{\dot{\gamma}x}}\right) & \cdots ② \end{cases}$$

伝搬定数：$\dot{\gamma} = \sqrt{(R + j\omega L)(G + j\omega C)}$

特性インピーダンス：$\dot{Z}_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

$$\dot{I}(0) = \frac{\dot{V}(0)}{\dot{Z}_0} \quad \cdots ③$$

$$\dot{V}(0) = E - 2\dot{Z}\dot{I}(0) \quad \cdots ④$$

①に$x = 0$を代入　$\dot{V}(0) = A \quad \cdots ⑤$

④に③,⑤を代入　$A = E - \dfrac{2\dot{Z}}{\dot{Z}_0}A$　　$A = \dfrac{\dot{Z}_0\dot{E}}{2\dot{Z} + \dot{Z}_0} \quad \cdots ⑥$

①,②に⑥を代入

$$\begin{cases} \dot{V}(x) = \dfrac{\dot{Z}_0\dot{E}}{2\dot{Z} + \dot{Z}_0}e^{-\dot{\gamma}x} \\ \dot{I}(x) = \dfrac{\dot{E}}{2\dot{Z} + \dot{Z}_0}e^{-\dot{\gamma}x} \end{cases}$$

分布定数回路（2）　《有限長線路》

①,②,④より　$A e^{-\dot{\gamma} l} + B e^{\dot{\gamma} l} = \dot{Z}_l \cdot \frac{1}{\dot{Z}_0}\left(A e^{-\dot{\gamma} l} - B e^{\dot{\gamma} l}\right)$

$$A\left(\dot{Z}_0 e^{-\dot{\gamma} l} - \dot{Z}_l e^{-\dot{\gamma} l}\right) + B\left(\dot{Z}_0 e^{\dot{\gamma} l} + \dot{Z}_l B e^{\dot{\gamma} l}\right) = 0$$

$$B = \frac{\dot{Z}_l e^{-\dot{\gamma} l} - \dot{Z}_0 e^{-\dot{\gamma} l}}{\dot{Z}_l e^{\dot{\gamma} l} + \dot{Z}_0 e^{\dot{\gamma} l}} A \qquad B = \rho e^{-2\dot{\gamma} l} A \quad \left(\rho = \frac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}\right) \quad \cdots⑤$$

点lにおける反射係数は③に⑤を代入して

$$\rho_l = \frac{B}{A} e^{2\dot{\gamma} l} = \frac{\rho e^{-2\dot{\gamma} l} A}{A} e^{2\dot{\gamma} l} = \rho = \frac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}$$

※$\dot{Z}_l = \dot{Z}_0$のとき、$\rho_l = 0$となり、反射は生じない。

伝搬方程式の解

入射波　反射波

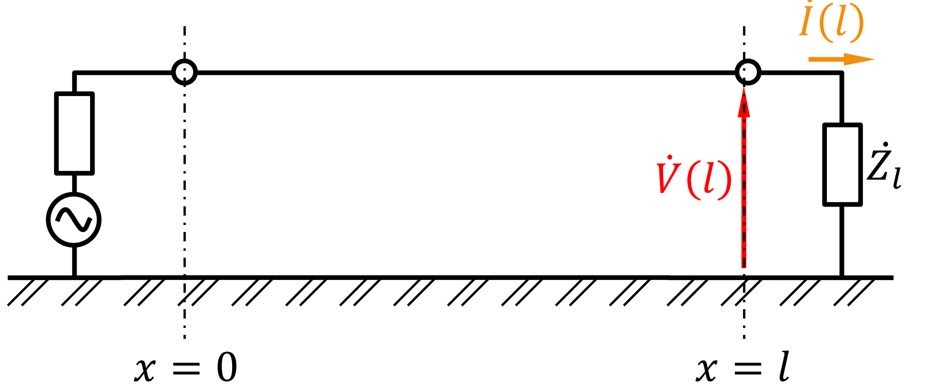
$$\begin{cases} \dot{V}(x) = A e^{-\dot{\gamma} x} + B e^{\dot{\gamma} x} & \cdots① \\ \dot{I}(x) = \frac{1}{\dot{Z}_0}\left(A e^{-\dot{\gamma} x} - B e^{\dot{\gamma} x}\right) & \cdots② \end{cases}$$

伝搬定数：$\dot{\gamma} = \sqrt{(R + j\omega L)(G + j\omega C)}$

特性インピーダンス：$\dot{Z}_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

反射係数：$\rho_x = \frac{\text{反射波}}{\text{入射波}} = \frac{B e^{\dot{\gamma} x}}{A e^{-\dot{\gamma} x}} = \frac{B}{A} e^{2\dot{\gamma} x}$　$\cdots③$

$\dot{V}(l) = \dot{Z}_l \dot{I}(l) \cdots④$



分布定数回路（２） 《入射波・反射波・透過波》

反射係数：$\rho_l = \dfrac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}$　　$\dot{V}_r = \rho_l \dot{V}_i$　　$\dot{I}_r = \rho_l \dot{I}_i$

入射波：$\dot{I}_i = \dfrac{\dot{V}_i}{\dot{Z}_0}$　　反射波：$\dot{I}_r = \dfrac{\dot{V}_r}{\dot{Z}_0}$　　透過波：$\dot{I}_t = \dfrac{\dot{V}_t}{\dot{Z}_l}$

$$\begin{cases} \dot{V}_t = \dot{V}_i + \dot{V}_r \\ \dot{I}_t = \dot{I}_i - \dot{I}_r \end{cases}$$

電圧は入射波と同位相で反射

電流は入射波の位相が反転して反射

電圧透過係数：$\lambda_{lV} = \dfrac{\text{透過波}}{\text{入射波}} = \dfrac{\dot{V}_t}{\dot{V}_i} = \dfrac{\dot{V}_i + \dot{V}_r}{\dot{V}_i} = \dfrac{\dot{V}_i + \rho_l \dot{V}_i}{\dot{V}_i} = 1 + \rho_l$

$$= 1 + \frac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0} = \frac{2\dot{Z}_l}{\dot{Z}_l + \dot{Z}_0}$$

電流透過係数：$\lambda_{lI} = \dfrac{\text{透過波}}{\text{入射波}} = \dfrac{\dot{I}_t}{\dot{I}_i} = \dfrac{\dot{I}_i - \dot{I}_r}{\dot{I}_i} = \dfrac{\dot{I}_i - \rho_l \dot{I}_i}{\dot{I}_i} = 1 - \rho_l$

$$= 1 - \frac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0} = \frac{2\dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}$$

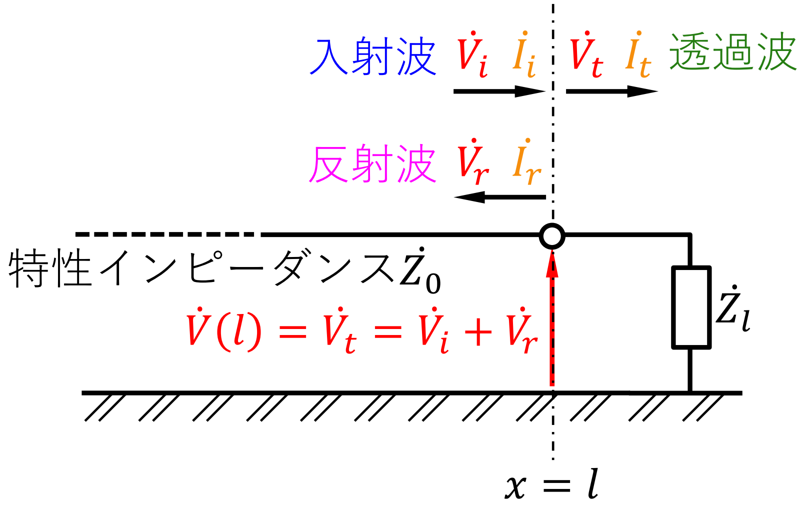
伝搬方程式の解

入射波　反射波

$$\begin{cases} \dot{V}(x) = Ae^{-\dot{\gamma}x} + Be^{\dot{\gamma}x} \\ \dot{I}(x) = \dfrac{1}{\dot{Z}_0}\left(Ae^{-\dot{\gamma}x} - Be^{\dot{\gamma}x}\right) \end{cases}$$

伝搬定数：$\dot{\gamma} = \sqrt{(R + j\omega L)(G + j\omega C)}$

特性インピーダンス：$\dot{Z}_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$

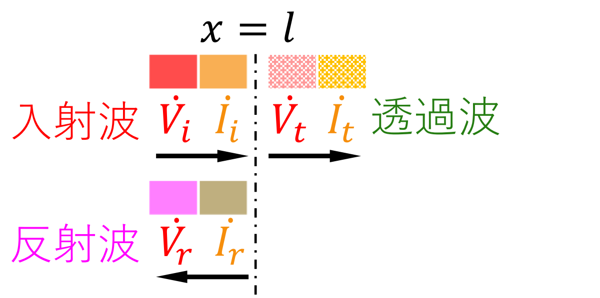


分布定数回路（2）　《反射・透過現象》

反射係数：$\rho_l = \dfrac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}$

電圧透過係数：$\lambda_{lV} = 1 + \rho_l$

電流透過係数：$\lambda_{lI} = 1 - \rho_l$

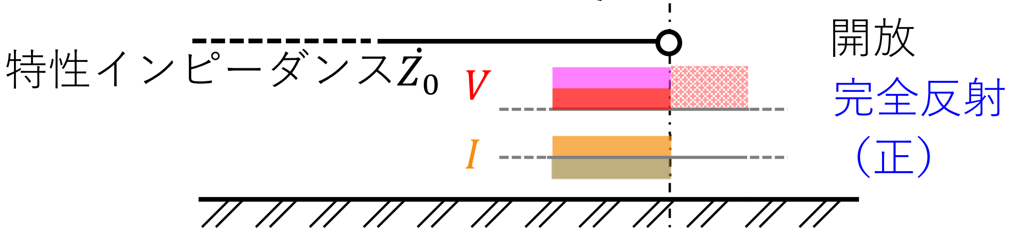


■開放　$\dot{Z}_l = \infty$

$$\rho_l = \frac{1-\frac{\dot{Z}_0}{\dot{Z}_l}}{1+\frac{\dot{Z}_0}{\dot{Z}_l}} = 1 \qquad \lambda_{lV} = 2 \qquad \lambda_{lI} = 0$$

$$\dot{V}_t = \dot{V}_i + \dot{V}_r = 2\dot{V}_i = \lambda_{lV}\dot{V}_i \qquad ※\dot{V}_r = \rho_l \dot{V}_i = \dot{V}_i$$

$$\dot{I}_t = \dot{I}_i - \dot{I}_r = 0 = \lambda_{lI}\dot{I}_i \qquad ※\dot{I}_r = \rho_l \dot{I}_r = \dot{I}_i$$

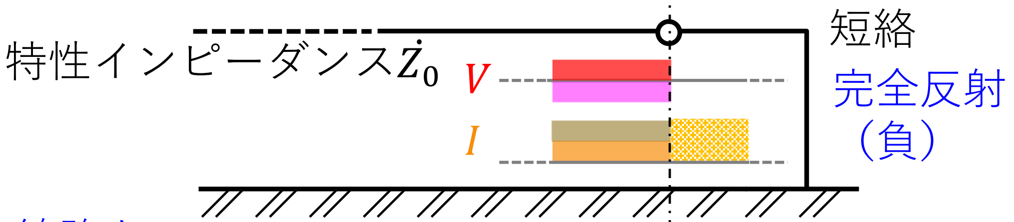


■短絡　$\dot{Z}_l = 0$

$$\rho_l = \frac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0} = -1 \qquad \lambda_{lV} = 0 \qquad \lambda_{lI} = 2$$

$$\dot{V}_t = \dot{V}_i + \dot{V}_r = 0 = \lambda_{lV}\dot{V}_i \qquad ※\dot{V}_r = \rho_l \dot{V}_i = -\dot{V}_i$$

$$\dot{I}_t = \dot{I}_i - \dot{I}_r = 2\dot{I}_i = \lambda_{lI}\dot{I}_i \qquad ※\dot{I}_r = \rho_l \dot{I}_r = -\dot{I}_i$$



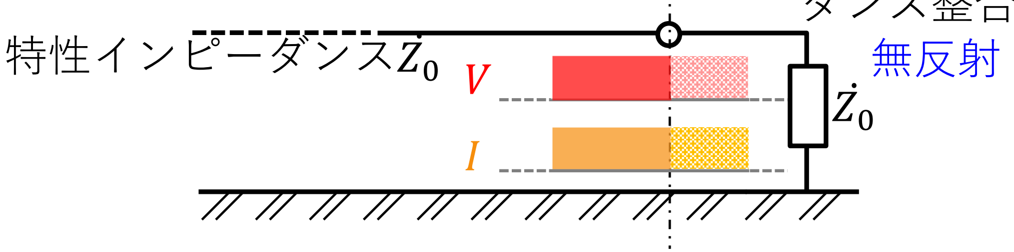
■インピーダンス整合　$\dot{Z}_l = \dot{Z}_0$　※特性インピーダンス$\dot{Z}_0$の線路を $\dot{Z}_l = \dot{Z}_0$なる負荷で終端すれば反射しない

$$\rho_l = \frac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0} = 0 \qquad \lambda_{lV} = 1 \qquad \lambda_{lI} = 1$$

$$\dot{V}_t = \dot{V}_i + \dot{V}_r = \dot{V}_i = \lambda_{lV}\dot{V}_i \qquad ※\dot{V}_r = \rho_l \dot{V}_i = 0$$

$$\dot{I}_t = \dot{I}_i - \dot{I}_r = \dot{I}_i = \lambda_{lI}\dot{I}_i \qquad ※\dot{I}_r = \rho_l \dot{I}_r = 0$$

インピーダンス整合



分布定数回路（２）　《透過波の式の導出１》

反射係数：$\rho_l = \dfrac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}$ ⋯①　$\dot{V}_r = \rho_l \dot{V}_i$ ⋯②　$\dot{I}_r = \rho_l \dot{I}_i$

入射波：$\dot{I}_i = \dfrac{\dot{V}_i}{\dot{Z}_0}$ ⋯③　反射波：$\dot{I}_r = \dfrac{\dot{V}_r}{\dot{Z}_0}$　透過波：$\dot{I}_t = \dfrac{\dot{V}_t}{\dot{Z}_l}$ ⋯④

$\dot{V}_t = \dot{V}_i + \dot{V}_r$ ⋯⑤　$\dot{I}_t = \dot{I}_i - \dot{I}_r$

①,②より　$\dot{V}_r = \dfrac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}\dot{V}_i$ ⋯⑥　$\dot{I}_r = \dfrac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}\dot{I}_i$

⑤,⑥より　$\dot{V}_t = \dot{V}_i + \dfrac{\dot{Z}_l - \dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}\dot{V}_i = \dfrac{2\dot{Z}_l}{\dot{Z}_l + \dot{Z}_0}\dot{V}_i$ ⋯⑦

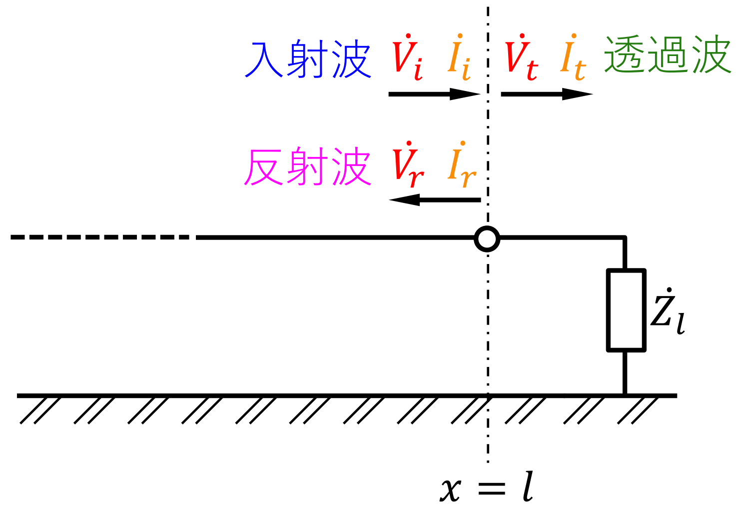
③,④,⑦より　$\dot{I}_t = \dfrac{\dot{V}_t}{\dot{Z}_l} = \dfrac{2\dot{Z}_l}{\dot{Z}_l + \dot{Z}_0}\dot{V}_i \cdot \dfrac{1}{\dot{Z}_l} = \dfrac{2}{\dot{Z}_l + \dot{Z}_0}\dot{V}_i = \dfrac{2\dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}\dot{I}_i$

電圧透過係数：$\lambda_{lV} = 1 + \rho_l = \dfrac{2\dot{Z}_l}{\dot{Z}_l + \dot{Z}_0}$

電流透過係数：$\lambda_{lI} = 1 - \rho_l = \dfrac{2\dot{Z}_0}{\dot{Z}_l + \dot{Z}_0}$

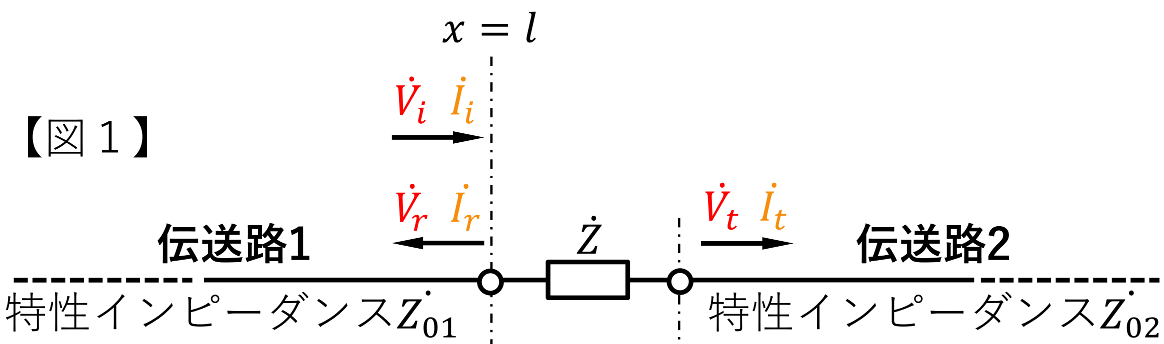
$\dot{V}_t = \lambda_{lV}\dot{V}_i$　$\dot{I}_t = \lambda_{lI}\dot{I}_i$

特性インピーダンス$\dot{Z}_0$の伝送路



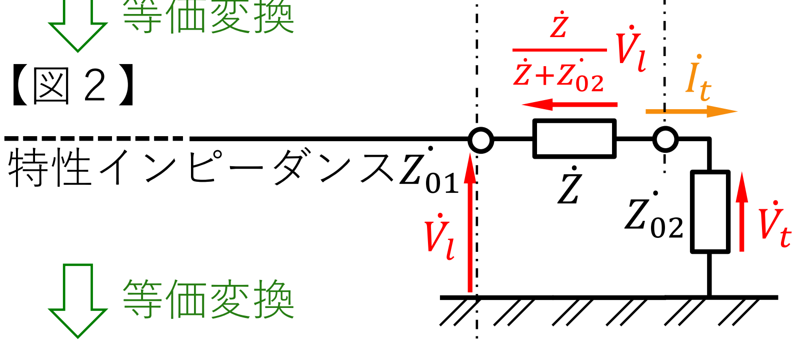
分布定数回路（2）　《透過波の式の導出２》

【図１】



⇩ 等価変換

【図２】

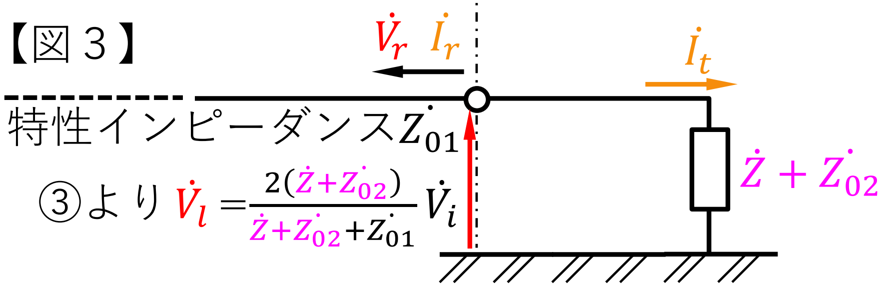


$$\dot{V}_t = \frac{\dot{Z}_{02}}{\dot{Z}+\dot{Z}_{02}}\dot{V}_l = \frac{\dot{Z}_{02}}{\dot{Z}+\dot{Z}_{02}}\cdot\frac{2(\dot{Z}+\dot{Z}_{02})}{\dot{Z}+\dot{Z}_{02}+\dot{Z}_{01}}\dot{V}_i$$

$$= \frac{2\dot{Z}_{02}}{\dot{Z}+\dot{Z}_{02}+\dot{Z}_{01}}\dot{V}_i \cdots ⑤$$

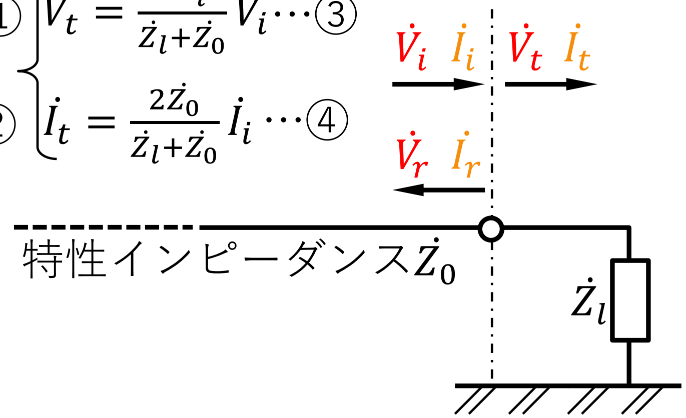
⇩ 等価変換

【図３】



③より $\dot{V}_l = \frac{2(\dot{Z}+\dot{Z}_{02})}{\dot{Z}+\dot{Z}_{02}+\dot{Z}_{01}}\dot{V}_i$

$$\begin{cases}\dot{V}_r = \frac{\dot{Z}_l-\dot{Z}_0}{\dot{Z}_l+\dot{Z}_0}\dot{V}_i \cdots ① \\ \dot{I}_r = \frac{\dot{Z}_l-\dot{Z}_0}{\dot{Z}_l+\dot{Z}_0}\dot{I}_i \cdots ②\end{cases} \quad \begin{cases}\dot{V}_t = \frac{2\dot{Z}_l}{\dot{Z}_l+\dot{Z}_0}\dot{V}_i \cdots ③ \\ \dot{I}_t = \frac{2\dot{Z}_0}{\dot{Z}_l+\dot{Z}_0}\dot{I}_i \cdots ④\end{cases}$$



【図A】

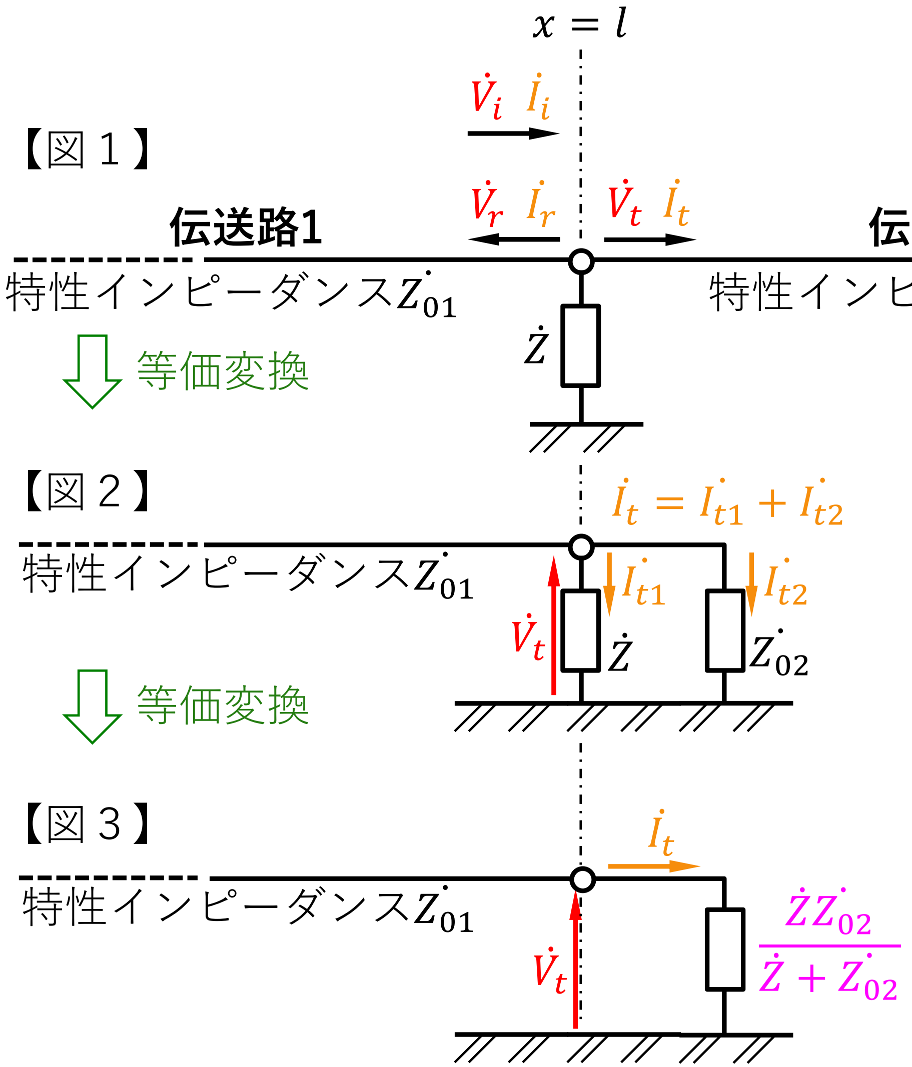
④,⑤より

$$\begin{cases}\dot{V}_t = \frac{2\dot{Z}_{02}}{\dot{Z}+\dot{Z}_{01}+\dot{Z}_{02}}\dot{V}_i \\ \dot{I}_t = \frac{2\dot{Z}_{01}}{\dot{Z}+\dot{Z}_{02}+\dot{Z}_{01}}\dot{I}_i\end{cases}$$

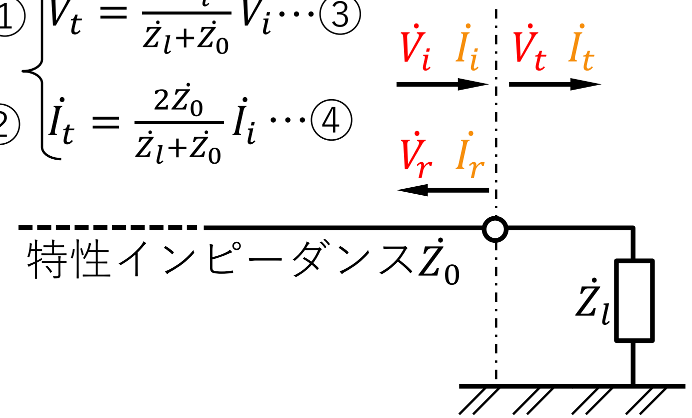
①,②より

$$\begin{cases}\dot{V}_r = \frac{\dot{Z}+\dot{Z}_{02}-\dot{Z}_0}{\dot{Z}+\dot{Z}_{02}+\dot{Z}_0}\dot{V}_i \\ \dot{I}_r = \frac{\dot{Z}+\dot{Z}_{02}-\dot{Z}_{01}}{\dot{Z}+\dot{Z}_{02}+\dot{Z}_{01}}\dot{I}_i\end{cases}$$

分布定数回路（２）　《透過波の式の導出３》



$$\begin{cases} \dot V_r = \dfrac{\dot Z_l - \dot Z_0}{\dot Z_l + \dot Z_0}\dot V_i \cdots ① \\ \dot I_r = \dfrac{\dot Z_l - \dot Z_0}{\dot Z_l + \dot Z_0}\dot I_i \cdots ② \end{cases} \quad \begin{cases} \dot V_t = \dfrac{2\dot Z_l}{\dot Z_l + \dot Z_0}\dot V_i \cdots ③ \\ \dot I_t = \dfrac{2\dot Z_0}{\dot Z_l + \dot Z_0}\dot I_i \cdots ④ \end{cases}$$



【図A】

③より $\dot V_t = \dfrac{2\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}}}{\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}} + \dot Z_{01}}\dot V_i = \dfrac{2\dot Z \dot Z_{02}}{\dot Z \dot Z_{01} + \dot Z \dot Z_{02} + \dot Z_{01}\dot Z_{02}}\dot V_i$

④より $\dot I_t = \dfrac{2\dot Z_{01}}{\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}} + \dot Z_{01}}\dot I_i = \dfrac{2\dot Z_{01}(\dot Z + \dot Z_{02})}{\dot Z \dot Z_{01} + \dot Z \dot Z_{02} + \dot Z_{01}\dot Z_{02}}\dot I_i$

①より $\dot V_r = \dfrac{\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}} - \dot Z_{01}}{\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}} + \dot Z_{01}}\dot V_i = \dfrac{\dot Z \dot Z_{02} - \dot Z \dot Z_{01} - \dot Z_{01}\dot Z_{02}}{\dot Z \dot Z_{01} + \dot Z \dot Z_{02} + \dot Z_{01}\dot Z_{02}}\dot V_i$

②より $\dot I_r = \dfrac{\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}} - \dot Z_{01}}{\frac{\dot Z \dot Z_{02}}{\dot Z + \dot Z_{02}} + \dot Z_{01}}\dot I_i = \dfrac{\dot Z \dot Z_{02} - \dot Z \dot Z_{01} - \dot Z_{01}\dot Z_{02}}{\dot Z \dot Z_{01} + \dot Z \dot Z_{02} + \dot Z_{01}\dot Z_{02}}\dot I_i$