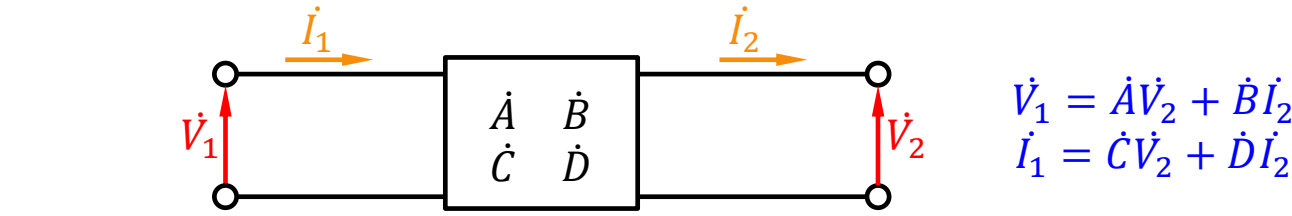
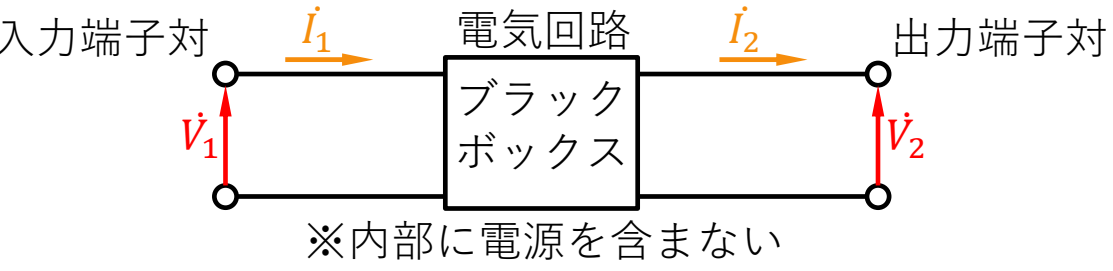


二端子対網 (1) 《K行列と四端子定数》

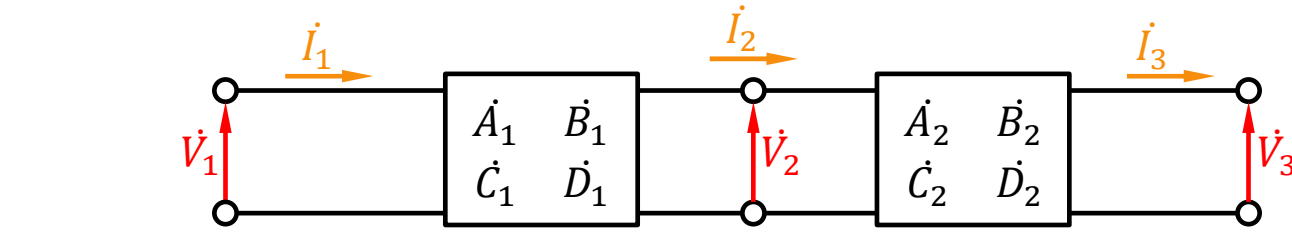


縦続行列 (K行列又はF行列)

$$\begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix} = K \begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix}$$

四端子定数 : A, B, C, D (Fパラメータ)

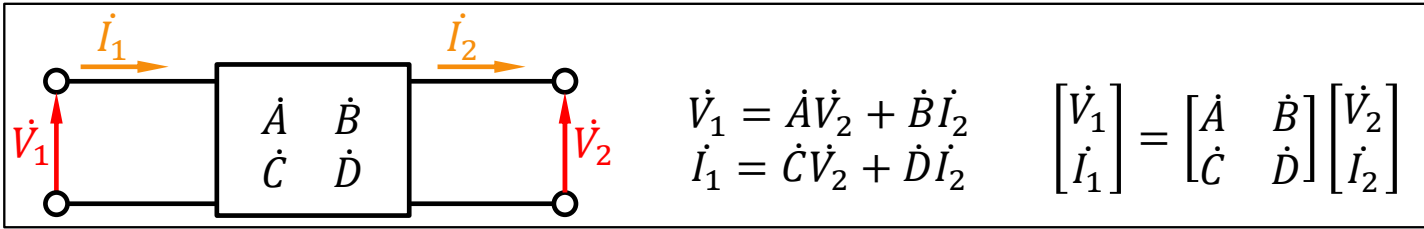
※複数の回路網を縦続接続したとき
縦続行列の積で全体を表せる



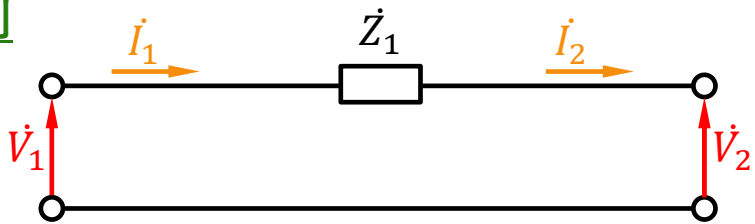
$$\begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix} \leftarrow \text{代入} \begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} \dot{V}_3 \\ \dot{I}_3 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix} &= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} \dot{V}_3 \\ \dot{I}_3 \end{bmatrix} \\ &= \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \begin{bmatrix} \dot{V}_3 \\ \dot{I}_3 \end{bmatrix} \end{aligned}$$

二端子対網 (1) 《K行列の合成1》

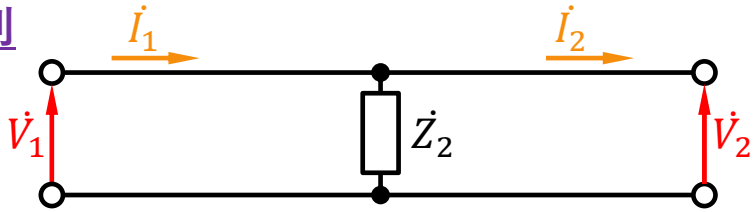


直列



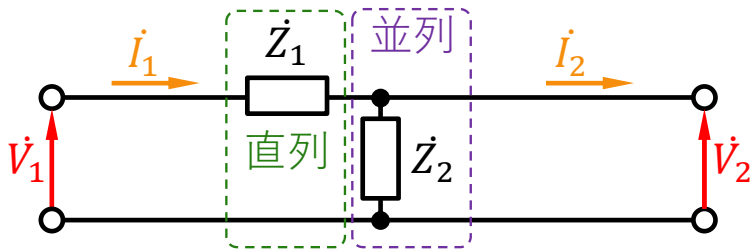
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \quad \begin{cases} \dot{V}_1 = \dot{V}_2 + Z_1 \dot{I}_2 \\ \dot{I}_1 = \dot{I}_2 \end{cases} \Rightarrow \begin{cases} \dot{V}_1 = 1 \cdot \dot{V}_2 + Z_1 \cdot \dot{I}_2 \\ \dot{I}_1 = 0 \cdot \dot{V}_2 + 1 \cdot \dot{I}_2 \end{cases}$$

並列



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \quad \begin{cases} \dot{V}_1 = \dot{V}_2 \\ \dot{I}_1 = \frac{\dot{V}_2}{Z_2} + \dot{I}_2 \end{cases} \Rightarrow \begin{cases} \dot{V}_1 = 1 \cdot \dot{V}_2 + 0 \cdot \dot{I}_2 \\ \dot{I}_1 = \frac{1}{Z_2} \cdot \dot{V}_2 + 1 \cdot \dot{I}_2 \end{cases}$$

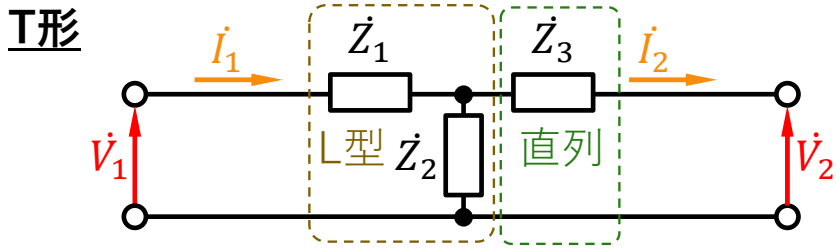
L形



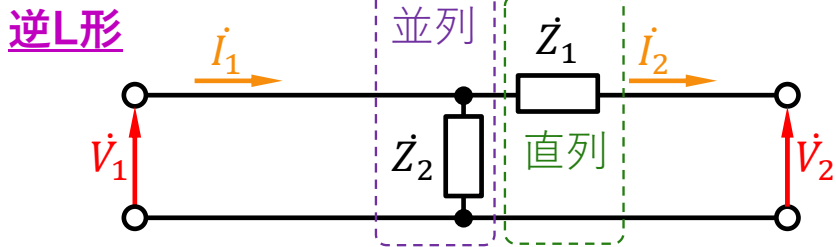
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

二端子対網 (1) 《K行列の合成2》

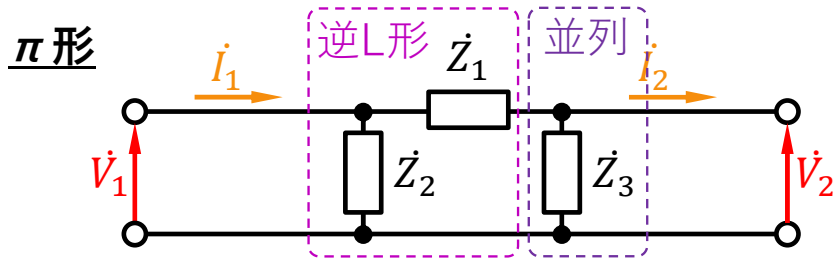
$$\text{直列} \begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix} \text{並列} \begin{bmatrix} 1 & 0 \\ \frac{1}{z_2} & 1 \end{bmatrix} \text{L形} \begin{bmatrix} 1 + \frac{z_1}{z_2} & z_1 \\ \frac{1}{z_2} & 1 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 + \frac{z_1}{z_2} & z_1 \\ \frac{1}{z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & z_3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{z_1}{z_2} & \frac{z_1 z_2 + z_2 z_3 + z_1 z_3}{z_2} \\ \frac{1}{z_2} & 1 + \frac{z_3}{z_2} \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{z_2} & 1 \end{bmatrix} \begin{bmatrix} 1 & z_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & z_1 \\ \frac{1}{z_2} & 1 + \frac{z_1}{z_2} \end{bmatrix}$$

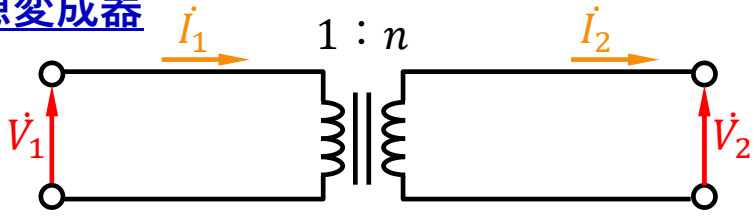


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z_1 \\ \frac{1}{z_2} & 1 + \frac{z_1}{z_2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{z_3} & 1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{z_1}{z_3} & z_1 \\ \frac{z_1 + z_2 + z_3}{z_2 z_3} & 1 + \frac{z_1}{z_2} \end{bmatrix}$$

二端子対網 (1) 《K行列の合成3》

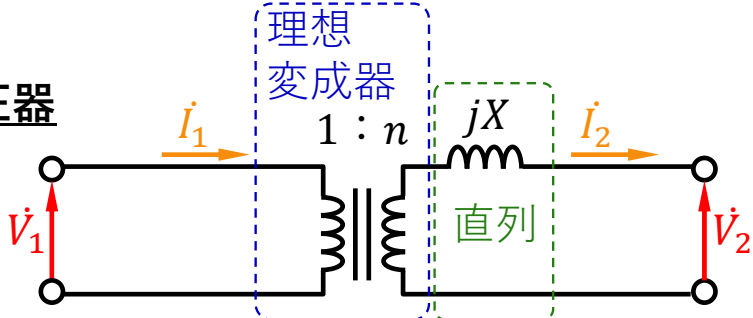
$$\text{直列} \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \text{並列} \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_2} & 1 \end{bmatrix} \text{L形} \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

理想変成器



$$\begin{bmatrix} \dot{A} & \dot{B} \\ \dot{C} & \dot{D} \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix} \quad \begin{cases} \dot{V}_1 = \frac{\dot{V}_2}{n} \\ \dot{I}_1 = n\dot{I}_2 \end{cases} \Rightarrow \begin{cases} \dot{V}_1 = \frac{1}{n} \cdot \dot{V}_2 + 0 \cdot \dot{I}_2 \\ \dot{I}_1 = 0 \cdot \dot{V}_2 + n \cdot \dot{I}_2 \end{cases}$$

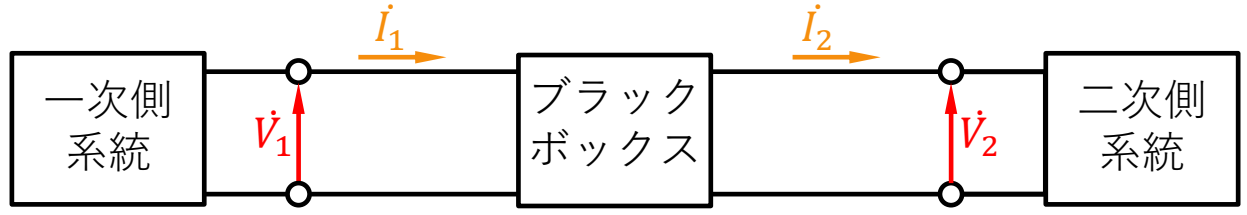
変圧器



$$\begin{bmatrix} \dot{A} & \dot{B} \\ \dot{C} & \dot{D} \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix} \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & \frac{jX}{n} \\ 0 & n \end{bmatrix}$$

二端子対網 (1) 《四端子定数の性質 1》

ブラックボックスの四端子定数



1. 二次側を開放 ($I_2 = 0$) して、 $\dot{V}_1, \dot{V}_2, \dot{I}_1$ を計測

$$A = \left. \frac{\dot{V}_1}{\dot{V}_2} \right|_{I_2=0} \quad C = \left. \frac{\dot{I}_1}{\dot{V}_2} \right|_{I_2=0}$$

$$\begin{cases} \dot{V}_1 = A\dot{V}_2 + B\dot{I}_2 \\ \dot{I}_1 = C\dot{V}_2 + D\dot{I}_2 \end{cases}$$

$$I_2 = 0 \Rightarrow \begin{cases} \dot{V}_1 = A\dot{V}_2 \Rightarrow A = \frac{\dot{V}_1}{\dot{V}_2} \\ \dot{I}_1 = C\dot{V}_2 \Rightarrow C = \frac{\dot{I}_1}{\dot{V}_2} \end{cases}$$

2. 二次側を短絡 ($\dot{V}_2 = 0$) して、 $\dot{V}_1, \dot{I}_1, \dot{I}_2$ を計測

$$B = \left. \frac{\dot{V}_1}{\dot{I}_2} \right|_{\dot{V}_2=0} \quad D = \left. \frac{\dot{I}_1}{\dot{I}_2} \right|_{\dot{V}_2=0}$$

$$\begin{cases} \dot{V}_1 = A\dot{V}_2 + B\dot{I}_2 \\ \dot{I}_1 = C\dot{V}_2 + D\dot{I}_2 \end{cases}$$

$$\dot{V}_2 = 0 \Rightarrow \begin{cases} \dot{V}_1 = B\dot{I}_2 \Rightarrow B = \frac{\dot{V}_1}{\dot{I}_2} \\ \dot{I}_1 = D\dot{I}_2 \Rightarrow D = \frac{\dot{I}_1}{\dot{I}_2} \end{cases}$$

二端子対網 (1)

《四端子定数の性質 2》

直列 $K = \begin{bmatrix} 1 & \dot{Z}_1 \\ 0 & 1 \end{bmatrix}$ $|K| = \begin{vmatrix} \dot{A} & \dot{B} \\ \dot{C} & \dot{D} \end{vmatrix} = \dot{A}\dot{D} - \dot{B}\dot{C} = 1 - 0 = 1$

並列 $K = \begin{bmatrix} 1 & 0 \\ \frac{1}{\dot{Z}_2} & 1 \end{bmatrix}$ $|K| = \dot{A}\dot{D} - \dot{B}\dot{C} = 1 - 0 = 1$

L形 $K = \begin{bmatrix} 1 + \frac{\dot{Z}_1}{\dot{Z}_2} & \dot{Z}_1 \\ \frac{1}{\dot{Z}_2} & 1 \end{bmatrix}$ $|K| = \dot{A}\dot{D} - \dot{B}\dot{C} = 1 + \frac{\dot{Z}_1}{\dot{Z}_2} - \frac{\dot{Z}_1}{\dot{Z}_2} = 1$

T形 $K = \begin{bmatrix} 1 + \frac{\dot{Z}_1}{\dot{Z}_2} & \frac{\dot{Z}_1\dot{Z}_2 + \dot{Z}_2\dot{Z}_3 + \dot{Z}_1\dot{Z}_3}{\dot{Z}_2} \\ \frac{1}{\dot{Z}_2} & 1 + \frac{\dot{Z}_3}{\dot{Z}_2} \end{bmatrix}$ $|K| = \dot{A}\dot{D} - \dot{B}\dot{C} = \left(1 + \frac{\dot{Z}_1}{\dot{Z}_2}\right)\left(1 + \frac{\dot{Z}_3}{\dot{Z}_2}\right) - \left(\frac{\dot{Z}_1\dot{Z}_2 + \dot{Z}_2\dot{Z}_3 + \dot{Z}_1\dot{Z}_3}{\dot{Z}_2}\right)\frac{1}{\dot{Z}_2}$
 $= 1 + \frac{\dot{Z}_3}{\dot{Z}_2} + \frac{\dot{Z}_1}{\dot{Z}_2} + \frac{\dot{Z}_1\dot{Z}_3}{\dot{Z}_2^2} - \left(\frac{\dot{Z}_1}{\dot{Z}_2} + \frac{\dot{Z}_3}{\dot{Z}_2} + \frac{\dot{Z}_1\dot{Z}_3}{\dot{Z}_2^2}\right) = 1$

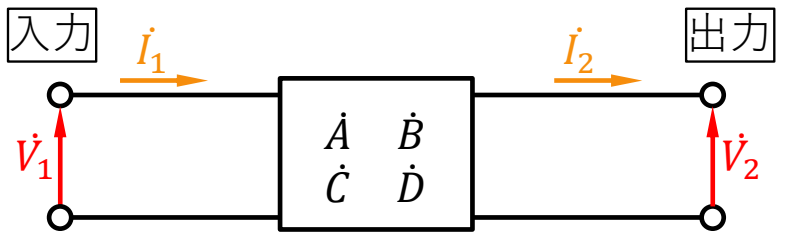
重要

四端子定数は、相反定理を満たす場合
 $|K| = \dot{A}\dot{D} - \dot{B}\dot{C} = 1$ となる

3つの定数が分かれば、残りの1つは
 $\dot{A}\dot{D} - \dot{B}\dot{C} = 1$ から求めることができる

二端子対網 (1)

《四端子定数の性質 3》

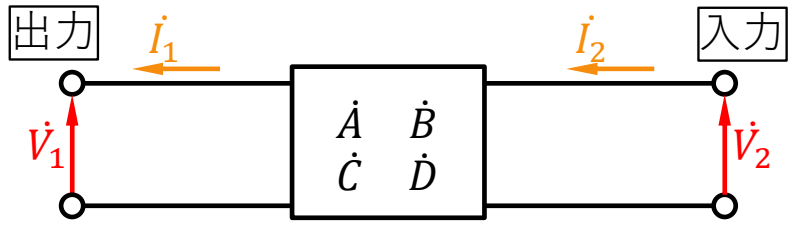


$$\begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} \dot{A} & \dot{B} \\ \dot{C} & \dot{D} \end{bmatrix} \begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix}$$

$$\ast K_1 = \begin{bmatrix} \dot{A} & \dot{B} \\ \dot{C} & \dot{D} \end{bmatrix}$$

↓ 入出力入替

↓ 入出力入替はDとAを入れ替えばよい



$$\begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} \dot{D} & \dot{B} \\ \dot{C} & \dot{A} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix}$$

$$\ast K_2 = \begin{bmatrix} \dot{D} & \dot{B} \\ \dot{C} & \dot{A} \end{bmatrix}$$

相反定理を満たさない場合：

$$\begin{bmatrix} \dot{V}_2 \\ \dot{I}_2 \end{bmatrix} = \frac{1}{|K|} \begin{bmatrix} \dot{D} & \dot{B} \\ \dot{C} & \dot{A} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{I}_1 \end{bmatrix}$$

$$\ast K_2 = \frac{1}{|K|} \begin{bmatrix} \dot{D} & \dot{B} \\ \dot{C} & \dot{A} \end{bmatrix}$$

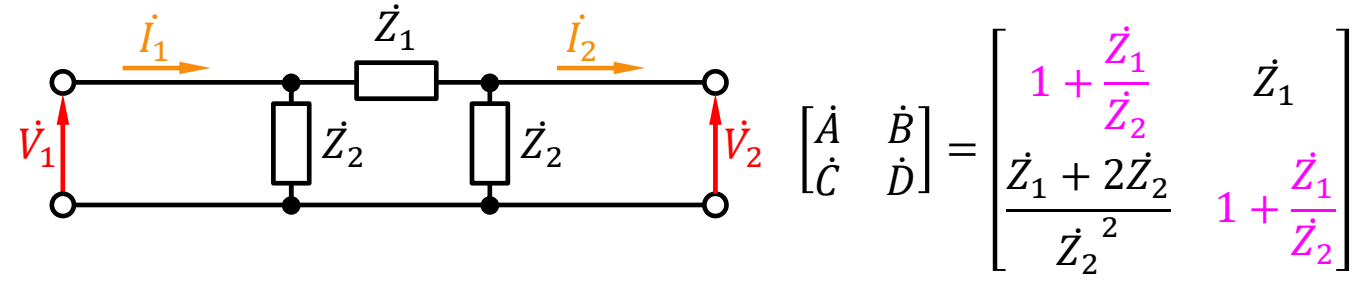
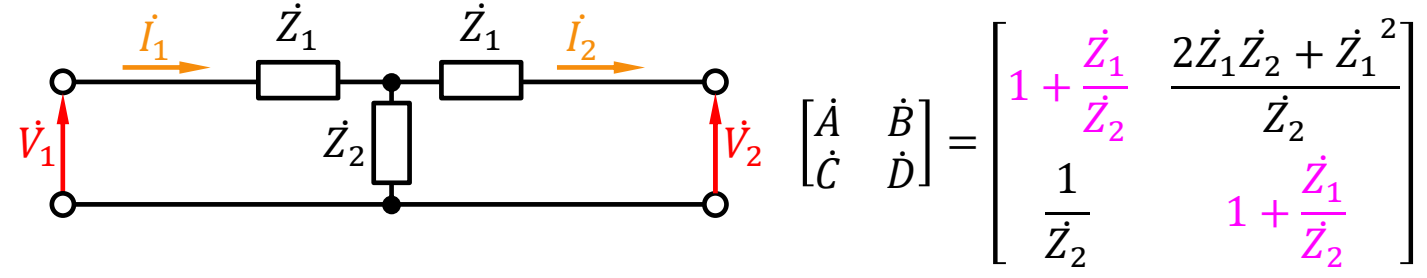
二端子対網 (1)

《四端子定数の性質 4》

対称二端子対網

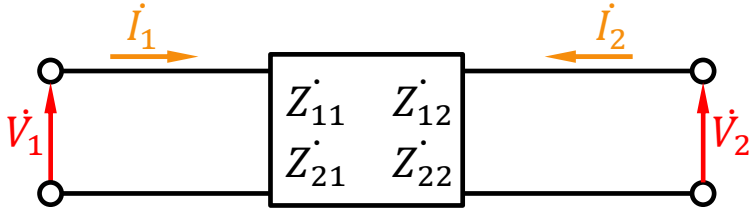
左右対称で入力側と出力側の区別がない二端子対網 のとき、 $A = D$ が成立する。

例)



二端子対網 (1)

《その他の二端子対網》

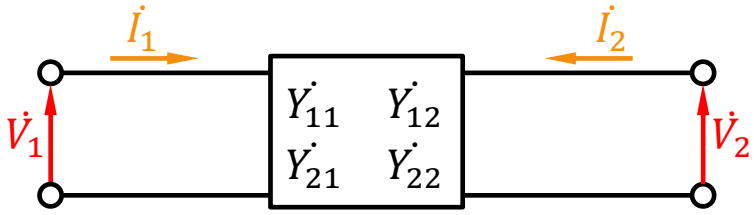


$$\begin{aligned} \dot{V}_1 &= Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{V}_2 &= Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{aligned}$$

インピーダンス行列 (Z行列)

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \mathbf{Z} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

$Z_{11}, Z_{12}, Z_{21}, Z_{22}$: Zパラメータ

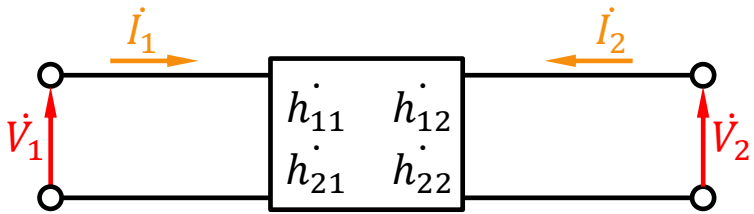


$$\begin{aligned} \dot{I}_1 &= Y_{11}\dot{V}_1 + Y_{12}\dot{V}_2 \\ \dot{I}_2 &= Y_{21}\dot{V}_1 + Y_{22}\dot{V}_2 \end{aligned}$$

アドミタンス行列 (Y行列)

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} = \mathbf{Y} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix}$$

$Y_{11}, Y_{12}, Y_{21}, Y_{22}$: Yパラメータ



$$\begin{aligned} \dot{V}_1 &= h_{11}\dot{I}_1 + h_{12}\dot{V}_2 \\ \dot{I}_2 &= h_{21}\dot{I}_1 + h_{22}\dot{V}_2 \end{aligned}$$

ハイブリッド行列 (H行列)

$$\begin{bmatrix} \dot{V}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{V}_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} \dot{I}_1 \\ \dot{V}_2 \end{bmatrix}$$

$h_{11}, h_{12}, h_{21}, h_{22}$: hパラメータ